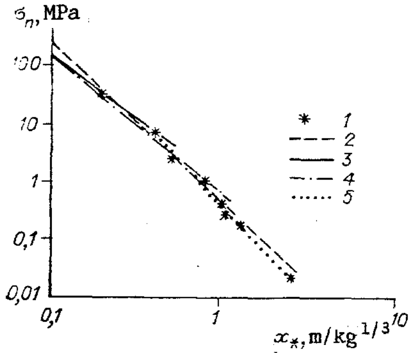

Fig. 1

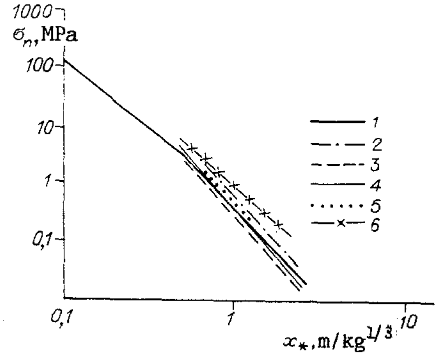

Fig. 2

LITERATURE CITED

1. V. D. Alekseenko, S. S. Grigoryan, A. F. Novgorodov, and G. V. Rykov, "Some experimental studies of soft soil dynamics," Dokl. Akad. Nauk SSSR, 133, No. 6 (1960).
2. G. M. Lyakhov, Fundamentals of Explosive Wave Dynamics [in Russian], Nedra, Moscow (1974).
3. S. S. Grigoryan, G. M. Lyakhov, and P. A. Parshukov, "Spherical explosive waves in soils as determined by stress and deformation measurements," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1977).
4. G. V. Rykov and A. M. Skobeev, Stress Measurements in Soil under Short-term Loading [in Russian], Nauka, Moscow (1978).
5. A. A. Vovk, B. V. Zamyshlyaev, L. S. Evterev, et al., Soil Behavior under Impulsive Loading [in Russian], Naukova Dumka, Kiev (1984).
6. S. I. Bodrenko, N. N. Gerdyukov, Yu. A. Krysanov, and S. A. Novikov, "Use of quartz pressure sensors in studies of shock wave processes," Fiz. Goreniya Vzryva, No. 3 (1981).
7. N. N. Gerdyukov, A. G. Ioilev, and S. A. Novikov, "Action of explosive loading on soft soil," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1992).

PRESSING THIN-WALLED TUBING FROM POWDERED MATERIAL

S. E. Aleksandrov and L. R. Vishnyakov

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The technique of pressing through a matrix (extrusion) has been widely used to produce bars and tubes from metallic powder [1]. Theoretical studies of this process in various apparatus have been carried out in [2-15].

A solution was obtained in [3] by means of the characteristic method without use of the continuity equation, not allowing determination of the density distribution without additional assumptions or experimental data. Approximate solutions of the equations of the plastic flow theory were given in [2, 6, 9]. In those studies it was assumed that the material followed Green's yield condition. In [2-6] the problem is solued by the method of planar cross sections. We consider flow in a matrix and a container. The system of ordinary differential equations obtained is solved numerically. In [5, 7-9] the finite element method was used to analyze nonsteady extrusion. A rigid-plastic model with cylindrical yield condition was obtained in [4]. In that study it was assumed that densification occurred only in the container, while in the matrix the material flowed in an uncompressed state. Extrusion without consideration of friction on the matrix walls was considered in [11]. Flow was assumed to be radial. It was shown in [12, 13] that in some cases the material must remain rigid in the container while compacting in the matrix. In those studies conditions were derived under which a flow was realized for the process of bar extrusion. Methods involving analysis of the energy of extrusion were used in [10, 14, 15]. In [10] the velocity field was assumed radial, while in [14] the planar section method was used. In [15] flow in both container and matrix was considered.

Extrusion of bimetallic tubes and bars (in which case the external material has the form of a tube) was considered in [9, 16-18]. The planar section method was used in [9, 16, 17], while the finite element method was used in [18].

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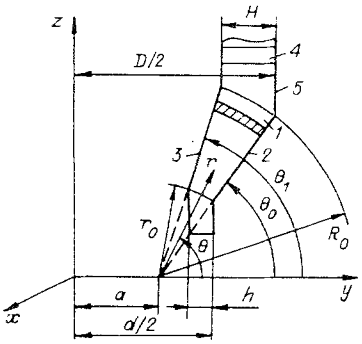

Fig. 1

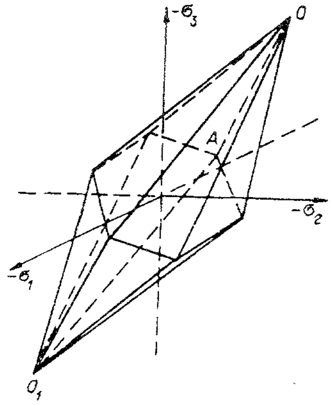

Fig. 2

Maintaining the rigid-plastic model the present study will analyze tube extrusion in a mandrel. The Hill method will be used to determine the equilibrium equations, which in contrast to the planar section method relates those equations to the chosen velocity field. At the same time a special coordinate system will allow production of a solution for quadratures for an arbitrary channel form in the meridional plane. It will be assumed that the material follows a meridional creep condition [12].

The material flow will be considered axisymmetric, i.e., the phenomenon of stability loss will not be considered. For flow in a converging channel of poured materials that phenomenon has been observed in experiment [19]. However experimental studies [20-23] under various conditions of metallic powder extrusion (for both planar and axisymmetric deformation) have not detected the stability loss phenomenon.

The solution will determine the density of the part and its distribution over the deformation hearth, as well as the stress-deformed state and pressing stress. Conditions will be determined under which steady state flow is possible, i.e., without densification in the container. It will be shown that under certain conditions the density of the part reaches a limiting value. The case under which the limiting density is reached within the deformation hearth will be investigated. The limiting pressure at which the process can be considered steady state will be determined. Attributes of the solution related to use of the piecewise-linear creep condition will be considered.

1. Determination of Part Density. A diagram of the pressing process is shown in Fig. 1 (1, material to be processed; 2, matrix; 3, mandrel; 4, plunger; 5, container). We introduce a coordinate system r, θ, φ. In Fig. 1 the arbitrary plane φ = const is shown. Let the material obey a pyramidal creep law (Fig. 2) [12]. In the general case of axisymmetric flow the stressed state may correspond to any edge or face of the creep surface and the flow regime must be determined from the solution. In analogy to the known solution of [13] we assume that the stressed state corresponds to the edge OA (Fig. 2). Here O is the peak of the creep pyramid in the semispace $\sigma < 0$ (where σ is the mean stress). The equations defining this edge have the form

$$\frac{\sigma_1-\sigma_2}{2\tau_s}-\frac{\sigma}{p_s}=1,\quad \frac{\sigma_1-\sigma_3}{2\tau_s}-\frac{\sigma}{p_s}=1, \tag{1.1}$$

where τ_s is the yield point for pure shear; p_s is the yield point for volume compression; σ_i are the main stresses (i = 1-3). We write the equation of the associated flow law as

$$\varepsilon_1=\lambda_1\left(\frac{1}{2\tau_s}-\frac{1}{3p_s}\right)+\lambda_2\left(\frac{1}{2\tau_s}-\frac{1}{3p_s}\right),\quad \varepsilon_2=-\lambda_1\left(\frac{1}{2\tau_s}+\frac{1}{3p_s}\right)-\lambda_2\frac{1}{3p_s},$$
$$\varepsilon_3=-\lambda_1\frac{1}{3p_s}-\lambda_2\left(\frac{1}{2\tau_s}+\frac{1}{3p_s}\right) \tag{1.2}$$

[ε_i are the main deformation rates (i = 1-3), $\lambda_1 \geqslant 0$, $\lambda_2 \geqslant 0$]. In the chosen coordinate system the deformation rates are written as:

$$\varepsilon_{rr}=\frac{\partial v_r}{\partial r},\quad \varepsilon_{r\theta}=\frac{1}{2}\left(\frac{\partial v_\theta}{\partial r}+\frac{1}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta}{r}\right),$$
$$\varepsilon_{r\varphi}=\frac{1}{2}\left[\frac{\partial v_\varphi}{\partial r}+\frac{1}{(a+r\cos\theta)}\left(\frac{\partial v_r}{\partial\varphi}-v_\varphi\cos\theta\right)\right],$$

$$\varepsilon_{\theta\varphi} = \frac{1}{2}\left[\frac{1}{(a + r\cos\theta)}\left(\frac{\partial v_\theta}{\partial\varphi} + v_\varphi \sin\theta\right) + \frac{1}{r}\frac{\partial v_\varphi}{\partial\theta}\right],$$
$$\varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial v_\theta}{\partial\theta} + v_r\right), \quad \varepsilon_{\varphi\varphi} = \frac{1}{(a + r\cos\theta)}\left(\frac{\partial v_\varphi}{\partial\varphi} + v_r\cos\theta - v_\theta\sin\theta\right) \tag{1.3}$$

(v_r, v_θ, v_φ are projections of the velocity in the corresponding coordinate directions).

Because of axial symmetry $v_\varphi = 0$. Since the velocity $v_\theta = 0$ on the matrix walls and the bar, for thin-wall parts we may take $v_\theta \equiv 0$ and $v_r = v(r)$. Then the components of the deformation rate tensor defined by Eq. (1.3) take on the form

$$\varepsilon_{rr} = \frac{dv}{dr}, \quad \varepsilon_{\theta\theta} = \frac{v}{r}, \quad \varepsilon_{\varphi\varphi} = \frac{v\cos\theta}{a + r\cos\theta}. \tag{1.4}$$

The remaining components are equal to zero, i.e., the coordinate directions are the main directions of the deformation rate tensor. Since the velocity projections $v < 0$, from Eq. (1.4) we obtain $\varepsilon_{\theta\theta} < 0$, $\varepsilon_{\varphi\varphi} < 0$. From Eq. (1.2) it follows that $\varepsilon_1 = \varepsilon_{rr}$. Eliminating the parameters λ_1 and λ_2 from Eq. (1.2) we obtain

$$\varepsilon_1 = -(\varepsilon_2 + \varepsilon_3)(3 - 2k_s)/(3 + 4k_s), \quad k_s = \tau_s/p_s. \tag{1.5}$$

Since $\varepsilon_2 + \varepsilon_3 = \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}$, it follows from Eqs. (1.4), (1.5) that

$$\frac{dv}{dr} + \frac{(3 - 2k_s)}{(3 + 4k_s)}\frac{(a + 2r\cos\theta)}{(a + r\cos\theta)}\frac{v}{r} = 0. \tag{1.6}$$

We will consider the steady state stage of the process. In view of the small thickness of the tube we take $\rho = \rho(r)$ (where ρ is relative density). Then for the chosen velocity field the continuity equation has the form

$$v\frac{d\rho}{dr} + \rho\left[\frac{dv}{dr} + \frac{(a + 2r\cos\theta)}{(a + r\cos\theta)}\frac{v}{r}\right] = 0. \tag{1.7}$$

Eliminating dv/dr from this equation, with the aid of Eq. (1.6) we find for the definition of ρ the equation

$$\frac{d\rho}{dr} + 6\rho\frac{(a + 2r\cos\theta)k_s}{r(a + r\cos\theta)(3 + 4k_s)} = 0,$$

to which we will apply the subregion method to average over thickness [24]. We divide the deformation hearth defined by the radii $r_0 = 1$ and R_0 into subregions defined by coordinate lines as shown in Fig. 1 (shaded region). Integrating Eq. (1.7) over θ, in view of the arbitrary nature of dr we obtain

$$r[a(\theta_1 - \theta_0) + r(\sin\theta_1 - \sin\theta_0)]\frac{d\rho}{dr} + \frac{6\rho k_s}{(3 + 4k_s)}[a(\theta_1 - \theta_0) + 2r(\sin\theta_1 - \sin\theta_0)] = 0. \tag{1.8}$$

For the process to occur in a steady state material of constant density ($\rho = \rho_0$) must appear at the matrix input. The solution of Eq. (1.8) for the condition $\rho = \rho_0$ at $r = R_0$ has the form

$$\int_\rho^{\rho_0}\frac{(3 + 4k_s)}{\rho k_s}d\rho = 6\int_r^{R_0}\frac{[a(\theta_1 - \theta_0) + 2r(\sin\theta_1 - \sin\theta_0)]}{r[a(\theta_1 - \theta_0) + r(\sin\theta_1 - \sin\theta_0)]}dr. \tag{1.9}$$

The dependence of k_s and τ_s on ρ can be written as [25]

$$k_s = (\sqrt{3}/2)[(1 - \rho)/\rho]^{1/2}, \quad \tau_s = \rho^3 k$$

(where k is the yield point for pure shear of the solid phase material). Then, integrating Eq. (1.9), we find the ρ distribution along the deformation hearth:

$$\arcsin(\rho^{1/2}) - \arcsin(\rho_0^{1/2}) + \ln(\rho/\rho_0)/\sqrt{3} = \frac{\sqrt{3}}{2}\ln\left\{\frac{R_0[a(\theta_1 - \theta_0) + R_0(\sin\theta_1 - \sin\theta_0)]}{r[a(\theta_1 - \theta_0) + r(\sin\theta_1 - \sin\theta_0)]}\right\}. \tag{1.10}$$

It is evident from this expression that at some $r = r_*$ the density reaches the limiting value $\rho = 1$. We define the quantity r_* from Eq. (1.10):

$$r_* = \frac{-a(\theta_1 - \theta_0) + [a^2(\theta_1 - \theta_0)^2 + 4c(\sin\theta_1 - \sin\theta_0)]^{1/2}}{2(\sin\theta_1 - \sin\theta_0)}. \tag{1.11}$$

Here $c = R_0[a(\theta_1 - \theta_0) + R_0(\sin\theta_1 - \sin\theta_0)]\exp[-(2/\sqrt{3})(\pi/2 - \arcsin(\rho_0^{1/2})) - \ln(\rho_0)/\sqrt{3}]$. If ρ does not reach the limiting value then the density of the part ρ_* is defined by Eq. (1.10) at $r = 1$.

We find the flow rate v from Eq. (1.6), in which we transform from integration over r to integration over ρ. As a result, with consideration of Eq. (1.7) we obtain

$$\frac{dv}{dr}-\frac{(3-2k_s)}{6\rho k_s}v=0. \tag{1.12}$$

We assume that at the input to the deformation hearth v = −1. Then the boundary condition for Eq. (1.12) has the form v = −1 at $\rho = \rho_0$. Integration of Eq. (1.12) yields

$$v=-\exp\left[\int\limits_{\rho_0}^{\rho}\frac{(3-2k_s)}{6\rho k_s}d\rho\right]. \tag{1.13}$$

With consideration of the expression used for k_s, we write

$$v=\exp\left[(2/\sqrt{3})\left(\arcsin(\rho^{1/2})-\arcsin\left(\rho_0^{1/2}\right)\right)-\ln(\rho/\rho_0)/3\right]. \tag{1.14}$$

To find the v distribution along the deformation hearth it is necessary to consider Eqs. (1.10) and (1.14) simultaneously.

Solutions of Eqs. (1.10) and (1.14) remain valid until the density at the deformation hearth reaches the value ρ = 1. This is insured by the condition $r_*\leqslant 1$ [r_* from Eq. (1.11)]. In the opposite case for $1\leqslant r\leqslant r_*$ the density ρ = 1, and v is found from Eq. (1.6) for k_s = 0. Averaging Eq. (1.6) over θ by the subregion method, we have

$$r\left[a(\theta_1-\theta_0)+r(\sin\theta_1-\sin\theta_0)\right]\frac{dv}{dr}+\left[a(\theta_1-\theta_c)+2r(\sin\theta_1-\sin\theta_0)\right]v=0 \tag{1.15}$$

with boundary condition $v = v_*$ at $r = r_*$, where v_* is the velocity value defined by Eq. (1.14) at ρ = 1:

$$v_*=-\exp\left[(2/\sqrt{3})\left(\pi/2-\arcsin\left(\rho_0^{1/2}\right)\right)+\ln(\rho_0)/3\right].$$

Integrating Eq. (1.15), we obtain

$$v=\frac{v_*r_*\left[a(\theta_1-\theta_0)+r_*(\sin\theta_1-\sin\theta_0)\right]}{r\left[a(\theta_1-\theta_0)+r(\sin\theta_1-\sin\theta_0)\right]}.$$

2. Determination of Pressing Pressure. We write the equilibrium equations using Hill's method [26]. In the case of the velocity field chosen, only a single equilibrium equation remains, which we write in the form

$$\int\limits_{\theta_0}^{\theta_1}\left[\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\left(\frac{\partial\tau_{r\theta}}{\partial\theta}+\sigma_r-\sigma_\theta\right)+\frac{1}{(a+r\cos\theta)}(\sigma_r\cos\theta-\sigma_\varphi\cos\theta-\tau_{r\theta}\sin\theta)\right]r(a+r\cos\theta)\,d\theta=0. \tag{2.1}$$

In view of the small wall thickness we take $\sigma_r=\sigma_r(r)=\sigma_1$, $\sigma_\theta=\sigma_\theta(r)=\sigma_2$, $\sigma_\varphi=\sigma_\varphi(r)=\sigma_3$. We express σ_θ and σ_φ in terms of σ_r, using the creep condition equations (1.1):

$$\sigma_\theta=\sigma_\varphi=\left[\sigma_r(3-2k_s)-6\tau_s\right]/(3+4k_s). \tag{2.2}$$

In Eq. (2.1) we transform from differentiation with respect to r to differentiation with respect to ρ with the aid of Eq. (1.7) and integrate over θ with consideration of Eq. (2.2). We apply a Coulomb friction law with constant coefficient of friction f, of the form $|\tau_{r\theta}||_{\theta=\theta_0}=|\tau_{r\theta}||_{\theta=\theta_1}=f|\sigma_\theta|$. By these transformations we obtain

$$\begin{gathered}6\rho k_s\left[\frac{a}{r}(\theta_1-\theta_0)+2(\sin\theta_1-\sin\theta_0)\right]\frac{d\sigma_r}{d\rho}+f\left(\frac{2a}{r}+\cos\theta_1+\cos\theta_0\right)\times\\ \times\left[\sigma_r(3-2k_s)-6\tau_s\right]-6(k_s\sigma_r+\tau_s)\left[\frac{a}{r}(\theta_1-\theta_0)+2(\sin\theta_1-\sin\theta_0)\right]=0.\end{gathered} \tag{2.3}$$

We define the quantity a/r from Eq. (1.10):

$$\frac{a}{r}=\frac{2(\sin\theta_1-\sin\theta_0)}{\theta_0-\theta_1+\left[(\theta_1-\theta_0)^2+4k_1a^{-2}\exp(k_2)(\sin\theta_1-\sin\theta_0)\right]^{1/2}}, \tag{2.4}$$

$$k_1=R_0\left[a(\theta_1-\theta_0)+R_0(\sin\theta_1-\sin\theta_0)\right],\quad k_2=-(2/\sqrt{3})\left(\arcsin\left(\rho^{1/2}\right)-\arcsin\left(\rho_0^{1/2}\right)+\ln(\rho/\rho_0)/\sqrt{3}\right).$$

Thus, Eq. (2.3) is linear in σ_r, and its solution with boundary condition $\sigma_r=\sigma_r^*$ for $\rho=\rho_1$ can be written as

$$\sigma_r = \exp\left[-\int_{\rho_1}^{\rho} p_1(t)\,dt\right]\left\{\sigma_r^* - \int_{\rho_1}^{\rho} Q_1(y)\exp\left[\int_{\rho_1}^{y} p_1(t)\,dt\right]dy\right\},$$

$$p_1(t) = \frac{f(2a/r + \cos\theta_1 + \cos\theta_0)(3-2k_s) - 6k_s[(\theta_1-\theta_0)a/r + 2(\sin\theta_1 - \sin\theta_0)]}{6tk_s[(\theta_1-\theta_0)a/r + 2(\sin\theta_1-\sin\theta_0)]}, \tag{2.5}$$

$$Q_1(y) = \frac{-\tau_s[f(2a/r + \cos\theta_1 + \cos\theta_0) + (\theta_1-\theta_0)a/r + 2(\sin\theta_1-\sin\theta_0)]}{yk_s[(\theta_1-\theta_0)a/r + 2(\sin\theta_1-\sin\theta_0)]}.$$

In the expressions for $p_1(t)$ and $Q_1(y)$ the value of the radius r must be expressed in terms of $t = \rho$ and $y = \rho$, respectively, from Eq. (2.4). We find the pressing pressure p from Eq. (2.5) at $\rho = \rho_0$ in the form $p = -\sigma_r(\rho_0)$.

Equation (2.5) defines the stress field for $r \geqslant r_*$. If $r_* \leqslant 1$. then Eq. (2.5) holds force over the entire deformation hearth. The boundary condition yields $\sigma_r^* = 0$, and the value of ρ_1 must be found from Eq. (1.10) for r = 1. If $r_* > 1$, then in the region $1 \leqslant r \leqslant r_*$ Eq. (2.1) takes on the form ($k_s = 0$)

$$\frac{1}{k}\frac{d\sigma_r}{dr} - \frac{f(2a/r + \cos\theta_1 + \cos\theta_0)\sigma_r/k}{a(\theta_1-\theta_0) + r(\sin\theta_1-\sin\theta_0)} + \frac{2[f(2a/r + \cos\theta_1 + \cos\theta_0) + 2(\sin\theta_1-\sin\theta_0)]r + 2a(\theta_1-\theta_0)}{r[a(\theta_1-\theta_0) + r(\sin\theta_1-\sin\theta_0)]} = 0. \tag{2.6}$$

A solution of Eq. (2.6) can be obtained from Eq. (2.5) by replacing ρ with r. Then

$$p_1(t) = -\frac{f(2a/t + \cos\theta_1 + \cos\theta_0)}{a(\theta_1-\theta_0) + t(\sin\theta_1-\sin\theta_0)},$$

$$Q_1(y) = \frac{[2f(2a/y + \cos\theta_1 + \cos\theta_0) + 4(\sin\theta_1-\sin\theta_0)]y + 2a(\theta_1-\theta_0)}{y[a(\theta_1-\theta_0) + y(\sin\theta_1-\sin\theta_0)]}k.$$

The boundary condition yields $\rho_1 = 1$, $\sigma_r^* = 0$ (ρ_1 is the coordinate r of the section of material exit from the matrix). In the interval $r_* \leqslant r \leqslant R_0$ we find the stress distribution from Eq. (2.5) for $\rho_1 = 1$, and σ_r^* from Eq. (2.6) at $r = r_*$.

When the pyramidal creep condition is used, it is necessary to verify satisfaction of the conditions $\lambda_1 \geqslant 0$, $\lambda_2 \geqslant 0$ and $0 \leqslant -\sigma \leqslant p_s$.

To verify the first two conditions we express λ_1 and λ_2 from Eq. (1.2) in the form

$$\lambda_1 = \frac{[2k_s\varepsilon_\varphi - (3+2k_s)\varepsilon_\theta]6\tau_s}{(3+2k_s)^2 - (2k_s)^2}, \quad \lambda_2 = \frac{[2k_s\varepsilon_\theta - (3+2k_s)\varepsilon_\varphi]6\tau_s}{(3+2k_s)^2 - (2k_s)^2}.$$

Hence, after integration we obtain

$$-a(3+2k_s)(\theta_1-\theta_0) - 3r(\sin\theta_1 - \sin\theta_0) \leqslant 0,$$
$$2k_s a(\theta_1-\theta_0) - 3r(\sin\theta_1 - \sin\theta_0) \leqslant 0.$$

Analysis of these inequalities shows that in the region $1 \leqslant r \leqslant r_*$ they are always satisfield, while in the region $r_* \leqslant r \leqslant R_0$ satisfaction of the inequality

$$\frac{a}{r} \leqslant \frac{3(\sin\theta_1-\sin\theta_0)}{2k_s(\theta_1-\theta_0)}$$

is necessary.

Using Eq. (2.4), we obtain a limitation upon the density in the form

$$\{(\theta_1-\theta_0)^2 + 4k_1a^{-2}\exp(k_2)(\sin\theta_1-\sin\theta_0)\}^{1/2} + \theta_0 - \theta_1 \geqslant 4k_s(\theta_1-\theta_0)/3. \tag{2.7}$$

To verify the inequality $0 \leqslant -\sigma \leqslant p_s$ we define the quantity $-\sigma$ as

$$-\sigma = -\frac{\sigma_r + \sigma_\theta + \sigma_\varphi}{3} = \frac{4\tau_s - 3\sigma_r}{3 + 4k_s}.$$

Hence $0 \leqslant 4\tau_s - 3\sigma_r \leqslant 3p_s + 4\tau_s$. Since $\sigma_r \leqslant 0$, then

$$|\sigma_r| \leqslant p_s. \tag{2.8}$$

To find the limits of applicability of the steady state process considered above it is necessary to compare the pressing force upon extrusion p with the stress at the beginning of material deformation in the container Q, i.e., upon precipitation of a ring in the closed press form. A theoretical analysis of this process was carried out, for example, in [27, 28]. However those studies used Green's creep condition, so that their results cannot be used in the present solution.

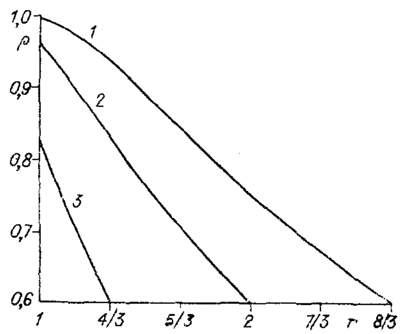

Fig. 3

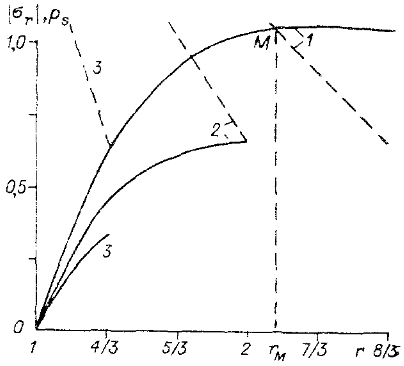

Fig. 4

We will note that a situation is possible in which only the portion of the material in the container located below the plunger is pressed. This is due to presence of a densification front during pressing in a closed matrix [27, 29]. In the given case the process will be a steady state until unpressed material is supplied to the beginning of the matrix. Consideration of container height is necessary for a general analysis of this process.

For the present we will limit ourselves to the situation in which densification in the container does not occur. If we take a one-dimensional velocity field in the container, as in the matrix, then from the creep condition and associated flow law it follows that the stressed state corresponds to the peak of the pyramid, i.e., $|\sigma| = p_s$. In this case $Q = p_s$ and the condition for transition of the material within the container into the plastic state coincides with condition (2.8), defining the possibility of using the above solution. Hence, steady state nature of the process is insured by the condition

$$|\sigma_r| < p_s \quad \text{for} \quad r = R_0. \tag{2.9}$$

3. Calculation Example. As an example we will consider the process of pressing a tube at f = 0.05 sec with initial density $\rho_0 = 0.6$. We will consider the dependence of the density distribution on reduction, for which we take the value $\delta = (H/h)$ and independent deformation hearth parameters (Fig. 1) as follows: D = 50 mm, d = 40 mm, $\theta_0 = \pi/4$. We calculate the remaining geometric characteristics from the expressions

$$\cos\theta_1 = \left[1 - \frac{2(H-h)}{D-d}\right]\cos\theta_0, \quad r_0 = \frac{h(D-d)}{2(H-h)\cos\theta_0},$$
$$R_0 = r_0\frac{H}{h}, \quad a = \frac{1}{2}d - r_0\cos\theta_0.$$

Wall thickness of the finished tube h = 3 mm.

Calculation results for density distribution are shown in Fig. 3 (δ = 8/3, 2, 4/3, lines 1-3). Satisfaction of condition (2.7) was verified in the calculation process. Together with Eq. (2.5) these dependences define the distribution of stress $|\sigma_r|$ along the deformation hearth (solid lines 1-3 of Fig. 4 for δ = 8/3, 2, 4/3, with dashed lines showing p_s distribution for corresponding δ values). It is evident from Fig. 4 that the solution at $\delta = 8/3$ is unsuitable, since for $r > r_M = 2.17$ condition (2.8) is unsatisfied. However, since the solution retains its force for $r \leqslant r_M$, it can be used for analysis of processes with a reduction $\delta < 2.17$ [at $\delta = 2.17$ condition (2.9) is violated], but in this case the density within the container cannot be specified arbitrarily, but must be determined from the solution (Fig. 3). For example, if $\delta = 2$, then $\rho_0 = 0.75$; if $\delta = 2$, then the solution with pre-specified initial density $\rho_0 = 0.6$ is limiting in the sense that the steady state flow under consideration will exist only at $\delta < 2$. This condition is satisfied by the case $\delta = 4/3$.

LITERATURE CITED

1. G. N. Morgun, "Processing of powdered material under pressure," in: Achievements in Science and Technology, Ser. Powder Metallurgy [in Russian], Vol. 3, VINITI, Moscow (1989).
2. G. L. Petrosyan, "Formation of porous tubes and bars," Dokl. Akad. Nauk ArmSSR, 14, No. 3 (1977).
3. V. E. Perel'man, Forming Powdered Materials [in Russian], Metallurgiya, Moscow (1979).
4. B. A. Druyanov and A. R. Pirumov, "Extrusion of porous materials," Vestn. Mashinostr., No. 9 (1980).

5. G. L. Petrosyan, G. G. Narseesyan, S. L. Malkhasyan, et al., "Packing of porous materials in rigid conical and cylindrical matrices," Poroshk. Metall., No. 5 (1982).
6. G. L. Petrosyan and G. V. Musaelyan, "Stressed state of porous axisymmetric billets during pressing," Poroshk. Metall., No. 11 (1984).
7. G. L. Petrosyan, G. V. Musaelyan, and Kh. L. Petrosyan, "Pressing of sintered porous material through a conical matrix," Poroshk. Metall., No. 3 (1985).
8. V. A. Osadchii, V. T. Zhadan, N. L. Gavrilov-Kryamichev, et al.,"Calculation of extrusion of a powdered billet through a conical matrix," Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 11 (1985).
9. G. L. Petrosyan, Plastic Deformation of Powdered Materials [in Russian], Metallurgiya, Moscow (1988).
10. J. Tirosh and D. Iddan, "Forming analysis of porous materials," Int. J. Mech. Sci., 31, No. 11/12 (1989).
11. M. E. Mear and D. Durban, "Radial flow of sintered powder metals," Int. J. Mech. Sci., 31, No. 1 (1989).
12. B. A. Druyanov, Applied Theory of Porous Body Elasticity [in Russian], Mashinostroenie, Moscow (1989).
13. S. E. Aleksandrov and B. A. Druyanov, "Steady-state extrusion of packed material," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1990).
14. A. A. Grigor'ev and V. N. Ivanov, "Deformation and packing of porous materials by direct pressing," Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 5 (1991).
15. G. L. Petrosyan, "Formation of bimetallic round bars with a porous core," Izv. Vyssh. Uchebn. Zaved., Mashinostr., No. 6 (1977).
16. L. A. Isaevich and T. A. Medvedeva, "Calculation of stress and density fields in powder pressing," in: Powder Metallurgy: Republican Inter-Institute Scientific Works Collection [in Russian], No. 15 (1991).
17. G. L. Petrosyan, "Formation of layered-porous bimetallic tubes," Izv. Akad. Nauk ArmSSR, Ser. Tekh. Nauk, 33, No. 6 (1980).
18. N. V. Mandkyan, S. G. Akbalyan, G. A. Tumakyan, et al., "Principles of bimetallic powdered material extrusion," Poroshk. Metall., No. 9 (1991).
19. A. F. Revuzhenko, S. B. Stazhevskii, and E. I. Shemyakin, "Plastic flow asymmetry in converging axisymmetric channels," Dokl. Akad. Nauk SSSR, 246, No. 3 (1979).
20. T. Tabata and S. Masaki, "Density distribution in extruded products of porous metal," J. Jpn. Soc. Tech. Plast., 17, No. 188 (1976).
21. T. Tabata, S. Masaki, and S. Shima, "Densification of green compacts by extrusion at low pressure," Int. J. Powder Metall. Powder Technol., 20, No. 1 (1984).
22. F. R. Karelyan and A. B. Savkin, "Powder compaction during hot deposit and extrusion," in: Plastic Deformation of Construction Materials [in Russian], Nauka, Moscow (1988).
23. D. G. Berghaus, R. J. Primas, and H. B. Peawek, "Strain analysis for extrusion of powder metals," Exp. Mech., 28, No. 3 (1988).
24. C. Fletcher, Numerical Methods Based on Galerkin's Method [Russian translation], Mir, Moscow (1988).
25. M. B. Shtern, "Theory of plasticity of porous bodies and compacted powders," in: Rheological Models and Deformation Processes in Porous Powdered and Composition Materials [in Russian], Naukova Dumka, Kiev (1985).
26. R. Hill, "Generalized method for analysis of metal processing," in: Mechanics, Collected Translations [Russian translation], No. 3 (1964).
27. S. E. Aleksandrov and B. A. Druyanov, "Pressing compacted materials in a closed pressform," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1990).
28. M. B. Shtern, G. G. Serdyuk, L. A. Maksimenko, et al., Phenomenological Theories of Powder Pressing [in Russian], Naukova Dumka, Kiev (1982).
29. L. M. Glukhov, V. G. Bakhtin, A. B. Kudrin, et al., "Methods for improving the quality of complex form powdered parts produced by pressing," Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 3 (1987).